Collapse Theory

Hong Kei Mui

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Abstract

In this project, we would go through the basic of the collapse theory (the Dynamical Reduction Models) which is a possible solution to solve the measurement problem in Quantum Mechanics and a theory to somehow bridge the micro and macroscopic world (as nowadays the microscopic world is described by QM and the macro one is described by classical physics, and do not have a satisfactory theory to connect two of them yet). We would see the overall effect of the collapse model on Quantum Mechanics by designing a simple Mach-Zehnder Interferometer Experiment. Through this experiment, we would indicate a possible way to design a real experiment to test the important parameters in the collapse model and so as the collapse model itself.

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Part I
Dynamical Reduction Models

As a physics theory, Quantum Mechanics was a great breakthrough in physics and is a theory that describes the microscopic world correctly. However, it also brought out some philosophical problem such as the famous Schrodinger's cat. The major problem of QM is the measurement problem, which appears mainly because the linear property of QM allows superposition of different quantum states.

1 QM Measurment

We consider an ideal measurement case here. We have a microsystem S and its observable quantum states s here, for example a particle and its spin. The quantum state has two different states, say spin up and down. The system S is associated with a macroscopic measurement apparatus B with an initial ready state B_0, which interacts with the microsystem S. The pointer of the apparatus has two position to indicate the two different states in S

$$|s_u\rangle \otimes |B_u\rangle \; and \; |s_d\rangle \otimes |B_d\rangle$$

If the system S starts with a single state $|s_u\rangle$ or $|s_d\rangle$, then there is no problem as the resultant macrostate is either $|B_u\rangle$ or $|B_d\rangle$, which means the pointer either points to up position or down position. However, if the system starts with a superposition state, for example $\frac{1}{\sqrt{2}}(|s_u\rangle + |s_d\rangle)$, then the measurement would be

$$\frac{1}{\sqrt{2}}(|s_u\rangle + |s_d\rangle) \otimes |B_0\rangle \to \frac{1}{\sqrt{2}}(|s_u\rangle \otimes |B_u\rangle + |s_d\rangle \otimes |B_d\rangle)$$

This means that there is a superposition of the macrostate, a superposition of the pointer pointing up and down position, which is obviously impossible. The wave packet suppression assumption of Quantum Mechanics which saying that the superposition quantum state would suppress to one of the quantum state with certain probability, $\frac{1}{2}$ for both states for the above example, when measurement is carried out. However, it is a rather embarrassing assumption as it means that the linear property of the quantum theory is broken in some level. The quantum theory then cannot fully explain what happens in the measurement and the theory breaks down in the macroscopic world [1].

Some solutions were proposed to solve this measurement problem such as hidden variable theory, which indicates Quantum Mechanics cannot describe the physical system completely and it is not a complete theory, and Dynamical Reduction Models, which modify Quantum Mechanics without changing the known results of the present quantum theory. Here we would consider the latter case, assuming that there is no hidden variables and Quantum theory is the one describing the whole system. One idea of the reduction models is the GRW theory, which basically is a theory to describe the random wave suppression process in quantum theory without changing the known results of Quantum Mechanics. One first consistent model of this is the Quantum Mechanics with Spontaneous Localizations model (QMSL) [1].

2 QMSL

One satisfactory theory of the dynamical reduction models is the Quantum Mechanics with Spontaneous Localizations (QMSL), which was first proposed by Ghirardi, Rimini and Weber. The aim of this model is to solve the measurement problem of the macroscopic system, which means to reduce the superposition of different macroscopic states without having many changes on the microscopic objects, so that it would not violate the known aspects of the quantum theory [1].

The QMSL model is based on 5 assumptions [1]:

1. All the particles of the system would experience a sudden spontaneous localization with a mean rate λ.

2. The system evolves with respect to the normal time dependent Schrodinger equation between each spontaneous localization.

3. One particle spontaneous localization is described as: $|\psi\rangle \rightarrow \frac{|\psi_x\rangle}{\||\psi_x\rangle\|}$ where $|\psi_x\rangle = L_x|\psi\rangle$ and $\||\psi_x\rangle\| = \langle\psi_x|\psi_x\rangle^{\frac{1}{2}}$, which L_x is a positive linear operator describing a localization of the particle at point x.

4. The probability density for a localization at point x is: $P(x) = \||\psi_x\|^2 = \langle\psi_x|\psi_x\rangle$ requiring that $\int dx[L_x]^2 = 1$.

5. The localization operator L_x have the following form:
$$L_x = (\frac{\alpha}{\pi})^{\frac{3}{4}}\, e^{-\frac{\alpha}{2}(\hat{x}-x)^2}$$
where $\hat{x}$ is the position operator of the particle, and α is relevant to the width of the localization Gaussian (the wave suppression length scale). As we are assuming a simple random process for the spontaneous localization, we use a normal Gaussian here to represent the process.

By using statistical operator ρ to describe the localization process, the process of localization is as follow [1]:

$$\begin{aligned}\rho = |\psi\rangle\langle\psi| \rightarrow \int dx P(x)\frac{|\psi_x\rangle\langle\psi_x|}{\langle\psi_x|\psi_x\rangle} &= \int dx|\psi_x\rangle\langle\psi_x| \\ &= \int dx L_x|\psi\rangle\langle\psi|L_x \\ &= T[\rho]\end{aligned}$$

which describes that starting with a statistical pure state and becomes a mixed state through the localization process.

Based on the above assumptions and the ordinary Schrodinger equation, we can derive the master equation governing the whole process of the system. Here we will use the statistical operator formalism, as it is convenient to use this form to investigate some useful physical properties of the model such as the time evolution of the means of dynamical variables [1]. Also we are considering one dimensional case here for simplicity.

First considering that in a time interval dt, there is a probability λdt for a localization to occur, and so the probability for no collapse occur would be $1-\lambda dt$. When a collapse happens, the statistical operator ρ transforms as above, so $\rho(t+dt) = \lambda dt T[\rho]$ for a localization. When there is no collapse, ρ just evolves according to the time dependent Schrodinger equation, and is as follow: $\rho(t+dt) = (1-\lambda dt)[\rho(t) - \frac{i}{\hbar}[H, \rho(t)]dt]$, where H is the normal Hamiltonian due to Schrodinger equation. Putting them together, ρ evolves as follow under this mechanism [1]:

$$\rho(t+dt) = (1-\lambda dt)[\rho(t) - \frac{i}{\hbar}[H, \rho(t)]dt] + \lambda dt T[\rho] \tag{1}$$

By rearranging the above equation and using first principle of differentiation, the equation becomes

$$\dot{\rho}(t) = -\frac{i}{\hbar}[H, \rho(t)] - \lambda[\rho(t) - T[\rho(t)]] \tag{2}$$

which is the master equation of this collapse mechanism. The equation decribes the quantum evolution of a one particle system with the random spontaneous localization processes, which is the main point of this model.

In coordinate representation, we have $\rho(x,y) = \langle x|\psi(t)\rangle\langle\psi(t)|y\rangle$ for a pure state, and so generally we have $\rho(x,y) = \langle x|\rho(t)|y\rangle$. According to assumption 1 and 5 for the statistical operator formalism, we found that $\langle x|T[\rho]|y\rangle = e^{-\frac{\alpha}{4}(x-y)^2 \langle x|\rho|y\rangle}$. Therefore the master equation in coordinate representation is as follow [1]

$$\begin{aligned}\dot{\rho}(x,y) &= -\frac{i}{\hbar}[H, \rho(x,y)] - \lambda[1 - e^{-\frac{\alpha}{4}(x-y)^2}]\rho(x,y) \\ &= \frac{i\hbar}{2m}[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}]\rho(x,y) - \lambda[1 - e^{-\frac{\alpha}{4}(x-y)^2}]\rho(x,y)\end{aligned} \tag{3}$$

From equation 3, we can see that the most important part is second term, which is the collapse process part. To see a rough behaviour for a macroscopic system, we can ignore the first term by assuming that the mass of the particle m is large. Focusing on the second term, we can see that the values of λ and α determine how the collapse process would affect the system.

To determine the values of these two parameters, there are some considerations to be taken into account. For example, the localization distance, which proportional to $\frac{1}{\alpha}$ here, is relevant to the atomic spacing of the lattice. Also by looking at the master equation of the model, we can see that a localization process would increase the energy of system. Therefore considering the situation of the present world such as the temperature of the universe, the mean rate of the occurence of collapse λ must not be too large.

About the possible values of this two parameters, it has been discussed in a paper by William Feldmann and Roderich Tumulka [2]. They have considered some experiments such as diffraction experiments and universal warming, and have determined the possible regions of the parameters from the experiment data. The following graph is quoted from their paper, showing the forbidden and possible regions of the values for λ and α.

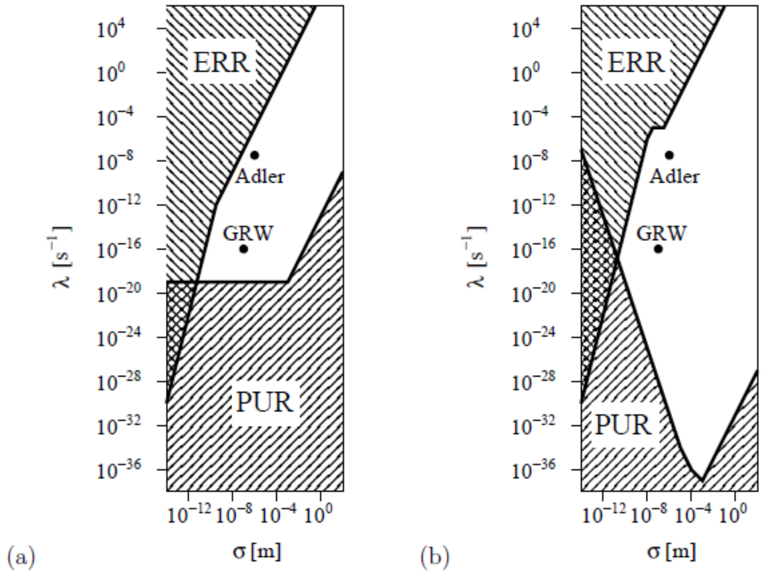

Figure 1: The shaded regions are the forbidden regions of the values of λ and σ, which is proportional to $\frac{1}{\alpha}$ here. (a) is corresponding to GRW theory and (b) is corresponding to CSL theory [2].

From the above graph we can see that the possible value of σ (the wave suppression length scale), which is proportional to $\frac{1}{\alpha}$, can be very large. Therefore it is not forbidden to assume that α is very small. By assuming that, equation 3 becomes

$$\dot{\rho}(x,y) = \frac{i\hbar}{2m}[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}]\rho(x,y) - \frac{D}{\hbar^2}(x-y)^2\rho(x,y) \tag{4}$$

where $D = \frac{\lambda\alpha\hbar^2}{4}$, which the two important parameters are included. One useful property of the density matrix is that the diagonal elements $\rho(x,x)$ of ρ describe the quantum statistical probability distribution of finding a particle at space point x, which is one reason why we would like to use the statistical operator formalism here.

Based on this property and equation 4, we can design a simple experiment which would be able to test the values of λ and α, and so as the effects of the model, given the value of α is very small. We would consider a simple model of Mach-Zehnder interferometer here. As we are assuming that the suppression length scale is large, in reality the setup of the interferometer would be very long.

3 CSL

The QMSL model seems to be a good one for the dynamical reduction models. However the model is not suitable for a system containing identical particles, since it does not preserve the symmetry property of the wavefunction for such a system. This problem is solved in the other model, the Continuous Spontaneous Localization Model (CSL), proposed by Ghirardi, Pearle, and Rimini. It is a model that replaces the discontinuous jumps in QMSL with a continuous stochastic process. To derive the master equation of the CSL model, we first look at the stochastic formalism of the QMSL model [1].

We first consider the Ito differential equation for a stochastic process $|\psi_B(t)\rangle$ [1]

$$d|\psi\rangle = [Cdt + AdB]|\psi\rangle \tag{5}$$

where C is the operator according to the normal Schrodinger evolution, A and B are set of operators $\{A_i\}$ and $\{B_i\}$ according to the random localization processes. B_i indicates a realization $B_i(t)$ of the Wiener processes (also known as the Brownian motions), which mean is zero and squared mean is proportional to t.

$$<< dB_i >>= 0 \; and \; << dB_i dB_j >>= \gamma\delta_{ij}dt$$

where λ is a real constant. The above raw process however does not conserve the norm of vectors. By using Ito calculus, we have the following [1]

$$\begin{aligned} d\||\psi(t)\rangle\|^2 &= \langle d\psi|\psi\rangle + \langle\psi|d\psi\rangle + << \langle\psi|d\psi\rangle >> \\ &= \langle\psi|(A + A^\dagger)|\psi\rangle dB + \langle\psi|(C + C^\dagger)|\psi\rangle dt + \langle\psi|A^\dagger A|\psi\rangle\gamma dt \end{aligned} \tag{6}$$

where $|d\psi\rangle \equiv d|\psi\rangle$. If the norm of the state vector $\psi_B(t)$ is one, then the probability of occurrence can just be the physical probability. However the norm is not one here, so we consider two normalized vectors here [1]

$$\begin{aligned} |\chi_B(t)\rangle &= \frac{|\psi_B(t)\rangle}{\||\psi_B(t)\rangle\|} \\ |\phi_B(t)\rangle &= \frac{|\psi_B(t)\rangle}{\||\psi_B(t)\rangle\|} \end{aligned}$$

$|\chi_B(t)\rangle$ has the probability according to vector $|\psi_B(t)\rangle$, and $|\phi_B(t)\rangle$ has the probability for $|\psi_B(t)\rangle$ times the squared norm $\||\psi_B(t)\rangle\|^2$. We choose the latter one as the physical probability, which is usually called cooked probability P_{cook}, and $|\chi_B(t)\rangle$ as the raw one. Now we have [1]

$$P_{cook}[B_i(t, t_0)] = P_{raw}[B_i(t, t_0)]\||\psi_B(t, t_0)\rangle\|^2 \tag{7}$$

Because of the linear property of the raw Ito differential equation 5 and the property of the Wiener process B_i. We can consider that the process from raw to physical is just performed at final time t_f or any number of time between initial time t_0 and t_f. We then can have the specialization of the above equation 7 for a infinitesimal time interval $(t_0, t_0 + dt)$, which gives us

$$P_{cook}[dB_i] = P_{raw}[dB_i][1 + \||\psi_B(t)\rangle\|^2] \tag{8}$$

By requiring that P_{cook} is one, we have $d << \||\psi\rangle\|^2 >>=<< d\||\psi\rangle\|^2 >>= 0$. From this result and equation 6, we have $C + C^\dagger = -\gamma A^\dagger A$. The raw process equation then becomes [1]

$$
\begin{aligned}
d|\psi\rangle &= [C^\dagger dt + AdB - \frac{\gamma}{2} A^\dagger A dt]|\psi\rangle \\
&= [-\frac{i}{\hbar} H dt + AdB - \frac{\gamma}{2} A^\dagger A dt]|\psi\rangle \qquad (9)
\end{aligned}
$$

which is the Ito linear equation for the stochastic process.

Alternatively, the above linear equation and the cooking probability statement 7 can be combined into one non linear stochastic differential equation for the state vector ϕ, which conserves the norm. This is an alternative way to express the QMSL in stochastic formalism.

By using the relation $C + C^\dagger = -\gamma A^\dagger A$ and equation 6, we have [1]

$$
d\||\psi\|^2 = \langle\psi|(A + A^\dagger)|\psi\rangle dB \qquad (10)
$$

and then from equation 8 we gets [1]

$$
\begin{aligned}
P_{cook}[dB_i] &= [1 + 2RdB]P_{raw}[dB_i] \\
R &= \frac{1}{2}\langle\psi|(A + A^\dagger)|\psi\rangle
\end{aligned}
$$

We have the normalized physical probability distribution P_{cook}, and we use a random variable dB' for this P_{cook}, with the following means

$$
<< dB'_i >>= 2\gamma R_i dt \ and \ << dB'_i dB'_j >>= \gamma\delta_{ij} dt
$$

then we have $dB' = dB + 2\gamma R dt$. B'_i has the same diffusion as the raw diffusion process B_i but shifted by $2R_i\gamma$. Now we can rewrite equation 9 for the two normalized vectors $|\chi(t)\rangle$ and $|\phi(t)\rangle$ by using equation 10 [1].

$$
\begin{aligned}
d|\chi\rangle &= [-\frac{i}{\hbar} H dt + (-\frac{1}{2}\gamma A^\dagger A - \gamma AR + \frac{3}{2}\gamma RR)dt + (A - R)dB]|\chi\rangle \qquad (11) \\
R &= \frac{1}{2}\langle\chi|(A + A^\dagger)|\chi\rangle
\end{aligned}
$$

The above equation conserves the norm and this property does not depend on the diffusion process B_i, which is different from the physical one with a shift.
The equation for the physical normalized vector is as follow, by replacing B_i with B'_i

$$
\begin{aligned}
d|\phi\rangle &= [-\frac{i}{\hbar} H dt + (-\frac{1}{2}\gamma A^\dagger A - \gamma AR + \frac{3}{2}\gamma RR)dt + (A - R)dB']|\phi\rangle \qquad (12) \\
R &= \frac{1}{2}\langle\phi|(A + A^\dagger)|\phi\rangle
\end{aligned}
$$

As we have the same diffusion B_i and B'_i, we can write

$$
\begin{aligned}
d|\phi\rangle &= [-\frac{i}{\hbar} H dt + (-\frac{1}{2}\gamma A^\dagger A - \gamma AR + \frac{3}{2}\gamma RR)dt + (A - R)dB]|\phi\rangle \qquad (13) \\
R &= \frac{1}{2}\langle\phi|(A + A^\dagger)|\phi\rangle
\end{aligned}
$$

which is a non linear stochastic differential equation for the physical process [1].

We can also write the stochastic differential equation in the statistiacal operator formalism. We have the density operator defined as [1]

$$
\begin{aligned}
\rho &= \int D[B_i] \frac{|\psi\rangle}{\||\psi\rangle\|} \frac{\langle\psi|}{\|\langle\psi|\|} P_{raw}[B_i] \||\psi\rangle\|^2 \\
&= << |\psi\rangle\langle\psi| >> P_{raw}
\end{aligned}
$$

where $\int D[B_i]$ is the functional integral corresponding to all realizations of the random process B_i. Using the stochastic differential equation and the above definition, and do the Ito calulus, we have [1]

$$
\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \gamma A \rho A^\dagger - \frac{\gamma}{2}\{A^\dagger A, \rho\} \tag{14}
$$

Now we can derive the master equation of the CSL model from the stochastic form of QMSL.

We start by choosing the proper functions of the creation and annihilation operators of particles for the operators A_i. Considering two operators, that satisfy the canonical commutation or anticommutation relations, $a^\dagger(y, s)$ and $a(y, s)$ for a particle at position x with a spin components s. Then the averaged density operator in general three dimension is defined as [1]

$$
N(x) = \sum_s \int d^3 y g(y - x) a^\dagger(y, s) a(y, s) \tag{15}
$$

where $g(x)$ is a spherically symmetric and positive real function peaked at $x = 0$, requiring that $\int d^3 g(x) = 1$, and so $\int dx N(x) = N$, where N is the number operator. We choose g to be the following form (comparing with the choice of localization operator in QMSL) [1]

$$
g(x) = (\frac{\alpha}{2\pi})^{\frac{3}{2}} e^{-\frac{\alpha}{2}x^2} \tag{16}
$$

in which $\alpha^{\frac{3}{2}}$ indicates the volume where the average is taken in the definition 15. The normalized eigenstate vectors of $N(x)$ are defined as usual [1]

$$
|q, s\rangle = \mathcal{N} a^\dagger(q_1, s_1) a^\dagger(q_2, s_2) ... a^\dagger(q_n, s_n)|0\rangle \tag{17}
$$

with normalization factor $\mathcal{N}$ and vacuum state $|0\rangle$.
The corresponding eigenvalues are $n(x) = \sum_{i=1}^{n} g(q_i - x)$.

With the above definition of $N(x)$, the Ito differential equation 9 then becomes [1]

$$
d|\psi\rangle = [-\frac{i}{\hbar} H dt + \int d^3 x N(x) dB(x) - \frac{\gamma}{2} \int d^3 x N^2(x) dt]|\psi\rangle \tag{18}
$$

with

$$
<< dB(x) >>= 0 \; and \; << dB(x) dB(y) >>= \gamma \delta^3(x - y) dt
$$

For the statistical operator formalism, equation 14 becomes

$$
\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \gamma \int d^3 x N(x) \rho N(x) - \frac{\gamma}{2} \int d^3 x \{N^2(x), \rho\} \tag{19}
$$

By using the definition 17, the above equation 19 then becomes

$$\frac{\partial}{\partial t}\langle q', s'|\rho(t)|q'', s''\rangle = -\frac{i}{\hbar}\langle q', s'|[H, \rho(t)]|q'', s''\rangle + \frac{\gamma}{2}\sum_{ij}[2G(q'_i - q''_j) - G(q'_i - q'_j) - G(q''_i - q''_j)]\langle q', s'|\rho(t)|q'', s''\rangle \tag{20}$$

where

$$G(q' - q'') = \int d^3x g(q' - x)g(q'' - x) = (\frac{\alpha}{4\pi})^{\frac{3}{2}} e^{-\frac{\alpha}{4}(q'-q'')^2}$$

which is the master equation for the CSL model [1].

Now if we only consider a one particle system, the master equation 20 becomes [1]

$$\frac{\partial}{\partial t}\langle q'|\rho(t)|q''\rangle = -\frac{i}{\hbar}\langle q'|[H, \rho(t)]|q''\rangle - \gamma(\frac{\alpha}{4\pi})^{\frac{3}{2}}[1 - e^{-\frac{\alpha}{4}(q'-q'')^2}]\langle q'|\rho(t)|q''\rangle \tag{21}$$

Rcall the master equation 3 we had in the above section for the QMSL model,

$$\dot{\rho}(x, y) = -\frac{i}{\hbar}[H, \rho(x, y)] - \lambda[1 - e^{-\frac{\alpha}{4}(x-y)^2}]\rho(x, y)$$

where $\rho(x, y) = \langle x|\rho(t)|y\rangle$. We can see that these two master equations are in fact the same one if we set $\gamma(\frac{\alpha}{4\pi})^{\frac{3}{2}} = \lambda$. Therefore, the effect of the collapse model on a single particle system is governed by the above master equation.

In the next part, we would focus on the approximated master equation 4 for a single particle system, from the QMSL section, and consider a simple thought experiment to see the effect of the collapse model (both QMSL and CSL for this one particle case) on the behaviour of a quantum interference.

Part II
Mach-Zehnder Interferometer Experiment

In this part, we would investigate how the quantum interference of particles is affected by the collapse model. We would consider a simple setup of Mach-Zehnder interferometer for one particle system. We are assuming that the quantum wave packets of the particle are separated rapidly at the beginning, so that there is no chance for the spontaneous localization to occur. Then bring them back together more slowly (so that the localization can occur). The setup is roughly like this:

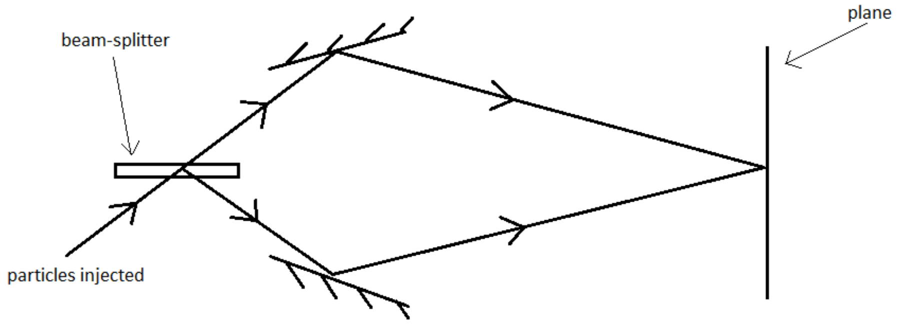

Figure 2: Simple setup of a Mach-Zehnder interferometer.

The first left part of the setup is designed to be short so that the separation process is very fast and short. The right part, where we would focus on and where everything happens, is much longer. Finally the two packets coincide on the plane. Based on Quantum Mechanics, we should see a interference pattern on the plane. By considering the spontaneous localization model, we will see how it would affect and change the result to be closer to the classical result.

For simplicity, we are considering one dimensional case here. Considering the two splitted waves move towards the plane as if there are two wave packets moving to each other in a one dimensional space.

1 Solution of The Approximated Master Equation

Based on the design of the experiment, we would set up an initial condition for solving equation 4. We consider an initial wave function that looks like this:

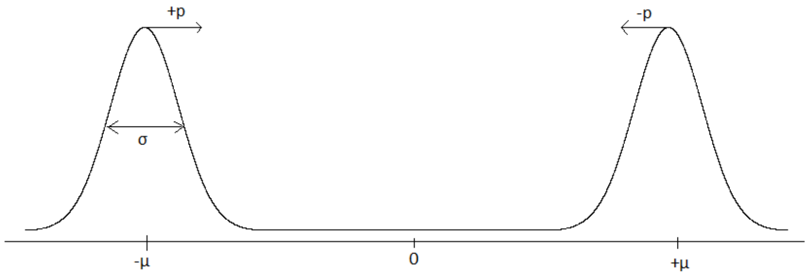

Figure 3: Initial wave function, where μ is the separation, p is the momentum and σ is the width of the Gaussian wave packets.

Two Gaussian peaks moving towards each other according to the two particles moving to the plane in the experiment setup. The wavefunction can be represented as:

$$\psi(x) = \frac{1}{\sqrt{2}}[\frac{1}{(2\pi\sigma^2)^{\frac{1}{4}}}e^{-\frac{(x-\mu)^2}{4\sigma^2}+\frac{ipx}{\hbar}} + \frac{1}{(2\pi\sigma^2)^{\frac{1}{4}}}e^{-\frac{(x+\mu)^2}{4\sigma^2}-\frac{ipx}{\hbar}}] \tag{22}$$

Then we can construct the initial density matrix as $\rho_0(x,y) = \psi^*(x)\psi(y)$. The full expression of the initial density matrix would be a linear combination of 4 terms of the following form:

$$\widetilde{\rho_0}(x,y) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\bar{x})^2}{4\sigma^2}}e^{-\frac{(y-\bar{y})^2}{4\sigma^2}}e^{\frac{i\bar{p}x}{\hbar}}e^{-\frac{i\bar{q}y}{\hbar}}$$

where $\bar{x}$, $\bar{y}$, $\bar{p}$ and $\bar{q}$ are with respect to different μ and p in $\psi(x)$ for the 4 terms. Then we solve equation 4 for an initial density matrix of the above form and we find the diagonal elements as follow:

$$\widetilde{\rho_t}(x,x) = \frac{1}{\sqrt{2\pi\sigma^2 J}}e^{-\frac{\bar{x}^2}{4\sigma^2}}e^{-\frac{\bar{y}^2}{4\sigma^2}}e^{\frac{\hbar t K^*}{2mJ}X^2}e^{\frac{\hbar t K}{2mJ}Y^2}e^{\frac{2Dt^3}{3mJ}XY} \tag{23}$$

$$X = -\frac{im}{\hbar t}x + \frac{\bar{x}}{2\sigma^2} + \frac{i\bar{p}}{\hbar}$$

$$Y = \frac{im}{\hbar t}x + \frac{\bar{y}}{2\sigma^2} + \frac{i\bar{q}}{\hbar}$$

$$K = \frac{2Dt^2}{3m\hbar} + \frac{\hbar t}{2m\sigma^2} - i$$

$$J = \frac{2Dt^3}{3m^2\sigma^2} + \frac{\hbar^2 t^2}{4m^2\sigma^4} + 1$$

The full expression of the diagonal elements solution is a linear combination of 4 terms of the above form, according to the initial wave function 22.

We want to investigate the behaviour of the interference pattern under the collapse mechanism. From equation 4, we can see that the effect of the spontaneous localization process mainly depends on the value of D. We only want to see the behaviour of the resultant pattern, so for simplicity we set $\hbar$, σ and m to be 1, and plot the solution in unit of those 3 parameters for all the following simulations. If D is zero, the solution simply describes a quantum interference pattern like the following graph.

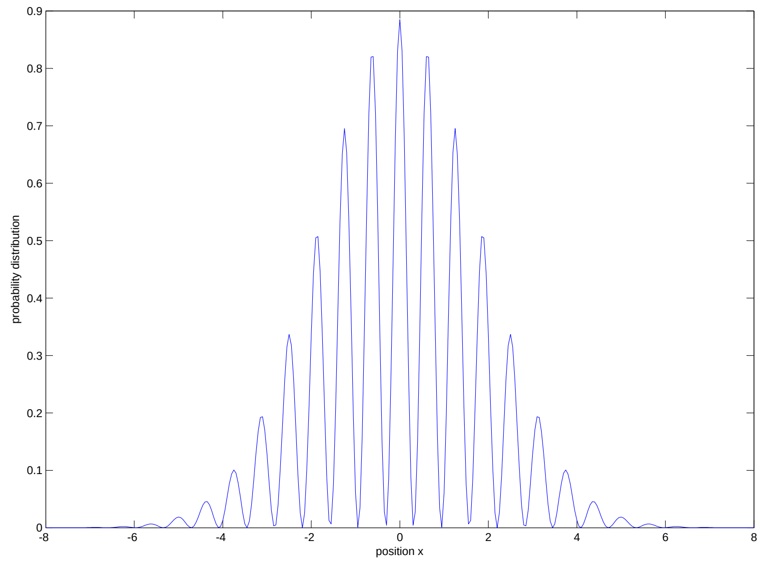

Figure 4: The normal quantum interference pattern, where $D = 0$, $\mu = 15$ and $p = 5$.

By putting in the value of D, we would see the effect of the collapse on the resultant pattern. The pattern is sensitive to the value of D. If the value is too small, the result would just be a normal Quantum Mechanics interference result. If it is too large, the result would just be a classical one without any quantum effect.

We want to set a reasonable value of D by now to see the behaviour, to do this we first choose a reasonable values of μ and p for the simulation. We chose the values of them so that the width of the Gaussian peak when it reaches the zero position point (the plane) would be less than the distance it travelled (the separation). Here we have chosen $\mu = 15$ and $p = 5$. Based on these two values, we chose a value of D to be 0.0025 by now. Then the resultant pattern would look like the following figure.

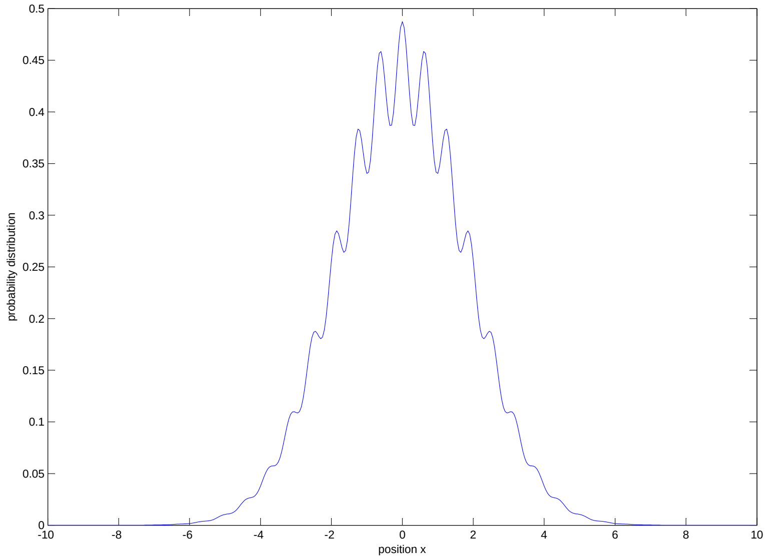

Figure 5: The resultant pattern on the plane, where $D = 0.0025$, $\mu = 15$ and $p = 5$.

From the above figure we can see how the behaviour would be under the collapse mechanism. The quantum interference pattern is being vanished and getting to a classical result for a macroscopic system.

Looking at the master equation, it is obvious that the resultant finge would be affected by the separation and time. Therefore we then want to investigate how μ and t would affect the pattern independently. By setting up a experiment to observe the change of the fringe according to the two parameters, we can check the correct values of D, and so as λ and α. In reality, it seems that changing the separation is better than changing the time in an experiment, as longer the time, harder to handle the whole system. That is why we consider a interferometer here, which allows us to change the separation. Now we would need a measure to observe the change of the fringe in the experiment. We would use fringe visibility as a measure here.

2 Fringe Visibility of The Interference Pattern

As we can see from figure 5, the interference pattern is losing as time evolves, which should be the case because there is more chances for the localization process to occur. The fringe visibility can help us to determine whether the pattern can still be distinguished as a quantum interference or not. Here we would use the normal definition of fringe visibility, which is $V = \frac{I_{max}-I_{min}}{I_{max}+I_{min}}$, where I_{max} is the probability of the central maximum and I_{min} is the probability of the first minimum of the pattern.

It is obvious to think that as time evolves, the fringe visibility would decrease because there is more chance for localization to occur and the fringe would tend to a classical result. In addition to time, we are interested in the effect of the separation μ, also from experiment point of view. Frome equation 4, we would expect that the increase of separation would reduce the fringe visibility, because the suppression term in the equation increases. The following graph describes the fringe pattern when two quantum packets concide on the plane for two different initial separations

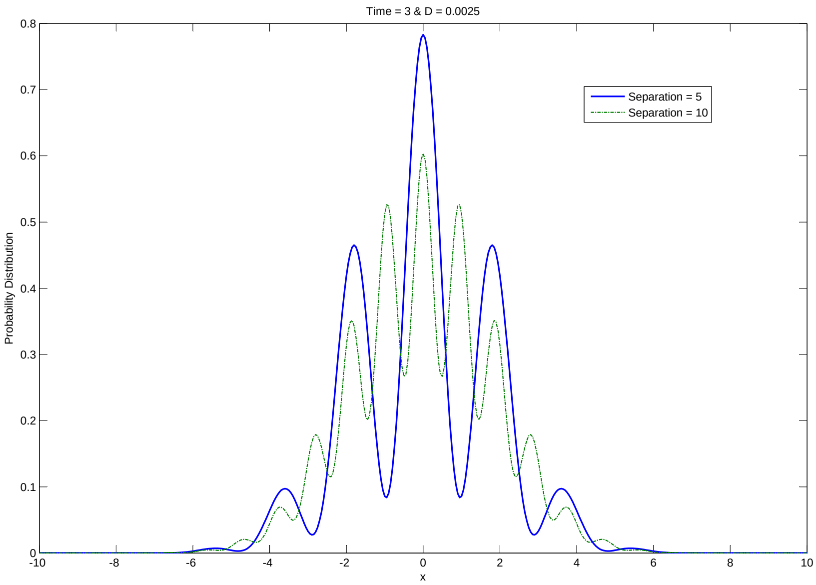

Figure 6: Fringe pattern for $\mu = 5$ and $\mu = 10$, where t is fixed to be 3 and $D = 0.0025$.

As expected, the fringe pattern tends to a classical result when increasing μ. Also from the graph we notice that the fringe spacing and the probability distribution keep changing for different values of μ, which is not surprising.

As the fringe spacing, so as the position of the first minimum, changes according to those parameters μ and t, we need to determine the position of the first minimum before we can look at the effects of changing μ and t. To do that, we first expand the full expression of solution 23. We find that all the complex components vanish, as expected, and left as the following form:

$$
\begin{aligned}
\rho_t(x,x) = & \frac{1}{\sqrt{2\pi\sigma^2 J}} e^{-\frac{\mu^2}{2\sigma^2}} \, e^{\frac{\hbar t}{2mJ}[\frac{Dt^2\mu^2}{3m\hbar\sigma^4} - \frac{4Dt^2}{3m\hbar}A^2 + \frac{\hbar t\mu^2}{4m\sigma^6} - \frac{\hbar t}{m\sigma^2}A^2 + \frac{2\mu}{\sigma^2}A]} \, e^{\frac{2Dt^3}{3mJ}[\frac{\mu^2}{a\sigma^4}+A^2]} \\
& + \frac{1}{\sqrt{2\pi\sigma^2 J}} e^{-\frac{\mu^2}{2\sigma^2}} \, e^{\frac{\hbar t}{2mJ}[\frac{Dt^2\mu^2}{3m\hbar\sigma^4} - \frac{4Dt^2}{3m\hbar}B^2 + \frac{\hbar t\mu^2}{4m\sigma^6} - \frac{\hbar t}{m\sigma^2}B^2 + \frac{2\mu}{\sigma^2}B]} \, e^{\frac{2Dt^3}{3mJ}[\frac{\mu^2}{a\sigma^4}+B^2]} \\
& + \frac{2}{\sqrt{2\pi\sigma^2 J}} e^{-\frac{\mu^2}{2\sigma^2}} \, e^{\frac{\hbar t}{2mJ}[\frac{Dt^2\mu^2}{3m\hbar\sigma^4} - \frac{2Dt^2}{3m\hbar}(A^2+B^2) + \frac{\hbar t\mu^2}{4m\sigma^6} - \frac{\hbar t}{2m\sigma^2}(A^2+B^2) + \frac{2\mu p}{\sigma^2\hbar}]} \, e^{\frac{2Dt^3}{3mJ}[-\frac{\mu^2}{a\sigma^4}+AB]} \\
& \times \cos\beta x
\end{aligned} \tag{24}
$$

where

$$
\begin{aligned}
\beta &= \frac{\hbar t}{2mJ}\left(\frac{4Dt\mu}{3\hbar^2} + \frac{\mu}{\sigma^4} + \frac{4mp}{\hbar^2 t}\right) - \frac{2Dt^2\mu}{3J\sigma^2\hbar} \\
A &= \frac{mx}{\hbar t} + \frac{p}{\hbar} \\
B &= \frac{mx}{\hbar t} - \frac{p}{\hbar}
\end{aligned} \tag{25}
$$

From equation 24 and 25, we can see that the first minimum is at the point when $\beta = \pm\pi$.

Now we can use the above results to see how the fringe visibility changes according to μ and t independently.

First of all we like to see the effect of changing t. We fixed the value of μ and using the fact that $\mu = pt$ to simulate the change of the fringe visibility. The following graph shows the change of fringe visibility according to time when D is fixed to be 0.0025.

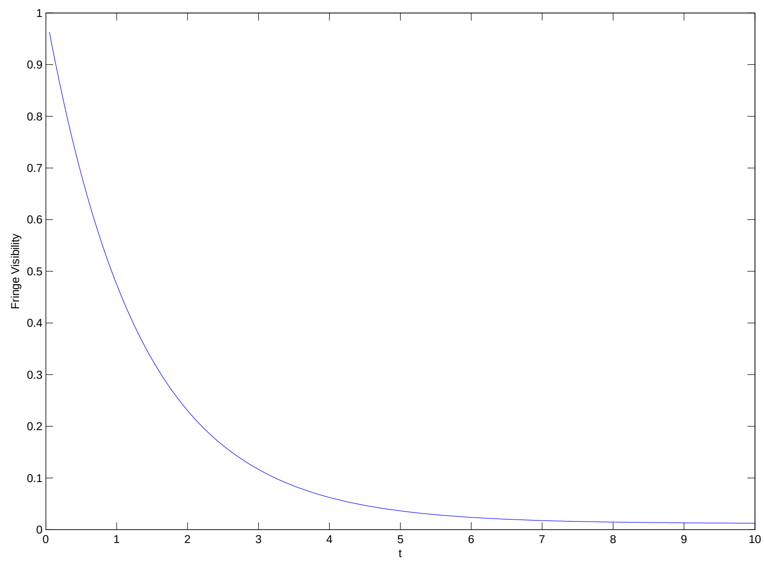

Figure 7: Fringe visibility according to time t, where μ is fixed to be 15 and $D = 0.0025$.

The result is as expected. It is also useful to see the differences between different values of D. The following graph indicates the change of fringe visibility for three different values of D.

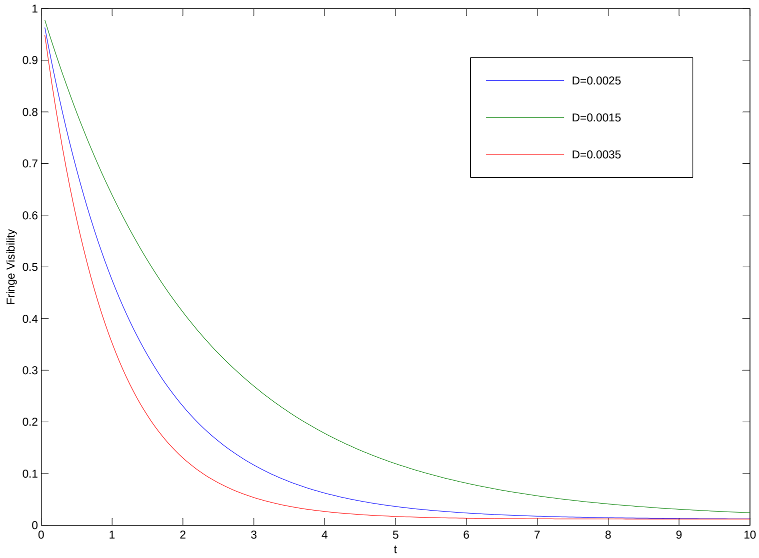

Figure 8: Fringe visibility according to time t for $D = 0.0015$, $D = 0.0025$ and $D = 0.0035$, where μ is fixed to be 15.

From the above figure, we can see that decreasing D would cause an increase of the overall fringe visibility and vise versa. The value of D governs the collapse process, from equation 4 we can see that decreasing D would reduce the effect of the localization process, so the simulation result of the solution agrees with the master equation. Frome this result, one can design an experiment to observe the fringe visibility and test the values of D, so as the two important parameters of the theory λ and α.

As written in the earlier section, from the experimental point of view, it is more convenient to use separation μ rather than time t. Because it is more difficult to control the system so that it is not disturbed in long time. The following graph indicates the change of fringe visibility according to the separation μ, with t fixed to be 3.

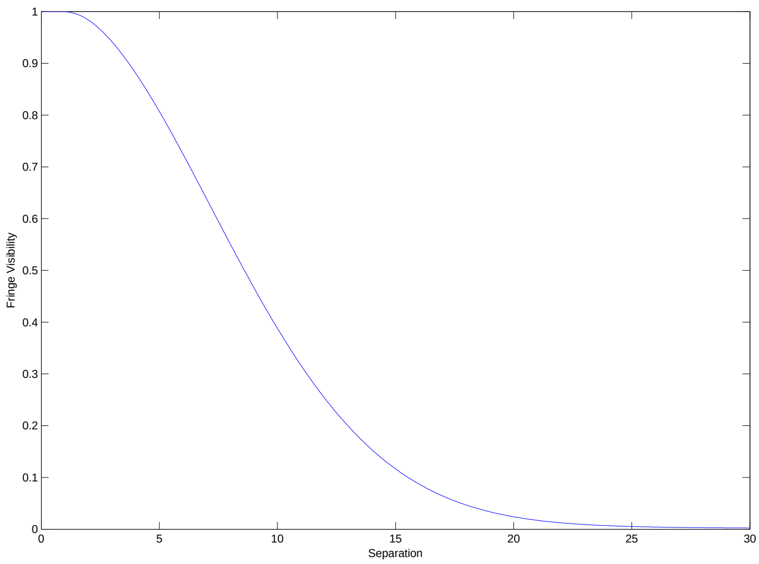

Figure 9: Fringe visibility according to separation μ for $D = 0.0025$, where t is fixed to be 3.

The fringe visibility decreases as separation increases, which agrees with the master equation. However, the fringe visibility at the small μ region should not be the case. Because if the separation is so samll and so that the fringing spacing is larger than the length scale of the Gaussian peak, when two packets concide on the plane, then the measure of fringe visibility is not a meaningful one in this case. The following graph shows the fringe pattern for small separation.

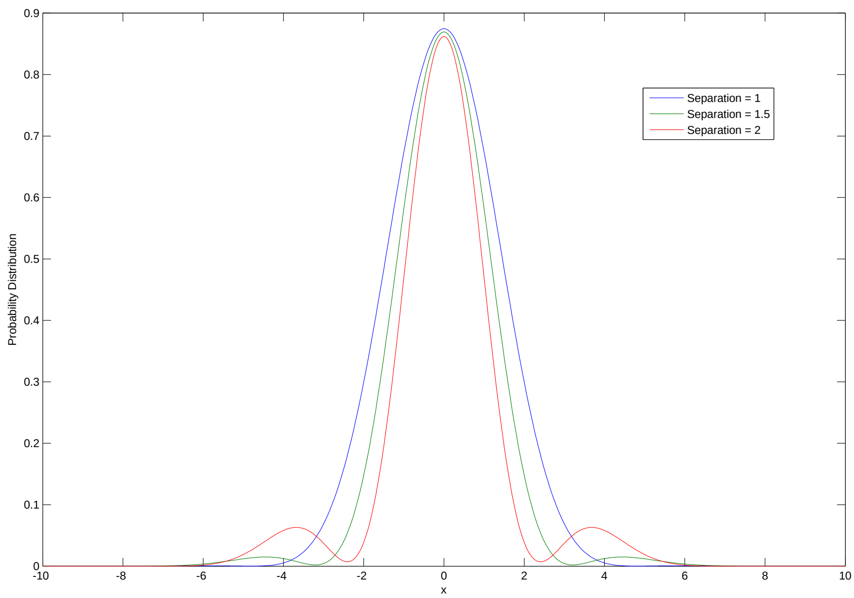

Figure 10: Fringe pattern for $\mu = 1$, $\mu = 1.5$ and $\mu = 2$, where $t = 3$ and $D = 0.0025$.

From the above figure we can see that the region lower than $\mu = 1$ is not good and should not be considered. Considering the change of fringe spacing according to D and experimental purpose, we choose $\mu = 2$ as a lower bond here.

Now we would see how fringe visibility change according to the value of D. The following graph indicates the fringe visibility for three different values of D.

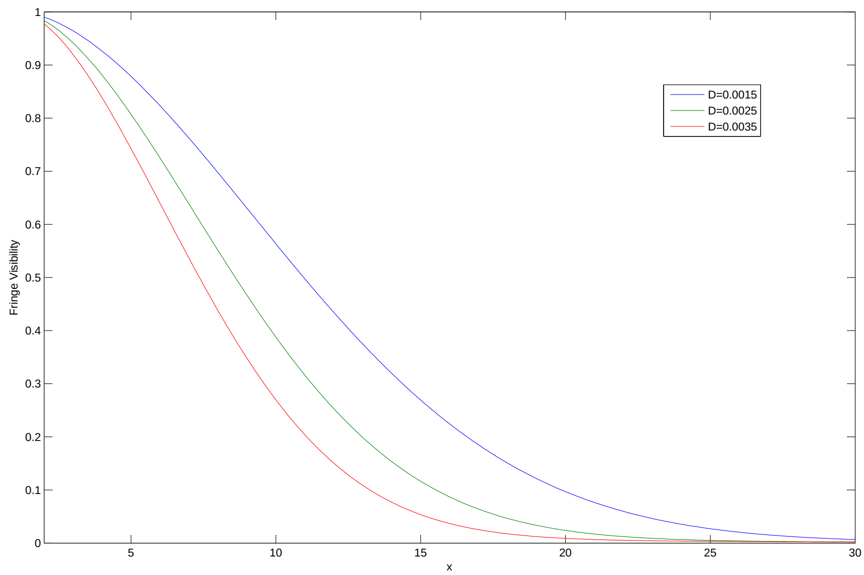

Figure 11: Fringe visibility according to separation μ for $D = 0.0015$, $D = 0.0025$ and $D = 0.0035$, where t is fixed to be 3.

The result agrees with equation 4, as the suppression rate would increase with the value of D and so the overall fringe visibility drops. From this result, one can design an experiment for changing separation to check the values of the two important parameters, λ and α in the spontaneous localization model. The above result is based on the solution of equation 4, which we assumed that the length scale of localization process is very large. Therefore an experiment based on this must be a setup with very large separation.

If we look at the second term, localization term, of the original master equation 3, we can see that the suppression rate would tend to a fixed value if the separation is large, given the value of λ and α is fixed. Comparing with equation 4, which the suppression rate increases with the separation, the fringe visibility result for the original master equation may not be the same as the above one. The fringe visibility may not drop as obvious for the original one.

As from the result of the paper by William Feldmann and Roderich Tumulka [2], small value of α is not forbidden by the experimental data. The approximation of the master equation can still be a good one. Looking at the overall behaviour of the fringe pattern in the above result, it seems that the result agrees with the original master equation to some extent and the approximation seems to be alright. An experiment with a large separation setup can be used to test the approximation of the master equation.

Conclusion

Here we designed a very simple thought experiment based on the single particle system solution of the master equation for QMSL and CSL. The overall behaviour of the resultant fringe pattern of a quantum interference experiment was simulated, taking into account the random spontaneous localization process. As the above results are based on an approximation for large length scale of the localization process. One can design a real Mach-Zehnder interferometer experiment with very long legs to test the behaviour of the resultant fringe, which would be a possible way to test whether the collapse model is a good theory to solve the measurement and connects the micro and macroscopic physical world. Also the experiment could be a test to find the values of the two important parameters, the mean rate and the length scale of the random localization process, in the theory.

Acknowledgements

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References

[1] Angelo Bassi and GianCarlo Ghirardi. Dynamical reduction models. arXiv:0302164v2, 2003.

[2] William Feldmann and Roderich Tumulka. Parameter diagrams of the grw and csl theories of wave function collapse. arXiv:1109.6579v2, 2011.